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Thermal Fluctuations and Hydrodynamic Effective Field Theory

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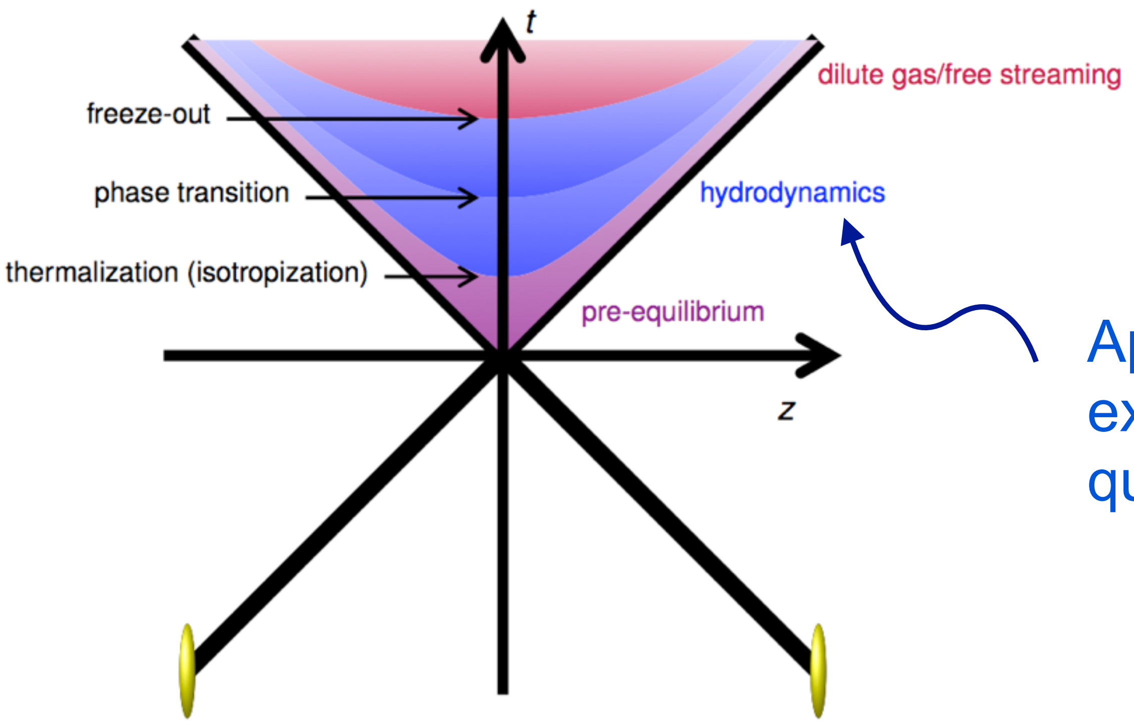
with P. Kovtun, to appear

[+ M. Harder, P. Kovtun - arXiv:1502.03076]

Hydrodynamics

Universal low energy “effective description” for scales » 1/T (with many dofs) over which an interacting system can achieve local thermal equilibrium

- parametrizes conserved currents via constitutive relations, and incorporates dissipation

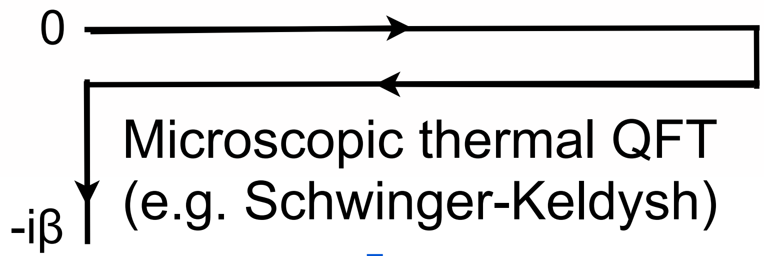


Applications, for example, to the quark-gluon plasma

Classical Hydrodynamics

all here

Hydrodynamic EFT



Microscopic thermal QFT
(e.g. Schwinger-Keldysh)

local thermal
equilibrium

$$\frac{\partial}{T} \ll 1$$

➠ What EFT incorporates both *dissipation* and *fluctuations?*

Hydrodynamic EFT

Classical hydrodynamics

Thermal fluctuations

Hydrodynamic Constitutive Relations

- Conservation laws:

$$\nabla_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho \qquad \nabla_\mu J^\mu = 0$$

In 3+1D, 5 eqns in 14 unknowns

- Constitutive relations:

$$\{\beta^\mu, \Lambda_\beta\} \quad \Rightarrow \quad T = \frac{1}{\sqrt{-\beta^2}}, \quad u^\mu = T\beta^\mu, \quad \mu = T(\Lambda_\beta + u^\mu A_\mu)$$

- Tensor basis

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P} P^{\mu\nu} + u^{(\mu} q^{\nu)} + \tau^{\mu\nu}$$

$$J^\mu = \mathcal{N} u^\mu + \nu^\mu$$

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- Near equilibrium ($t_{\text{therm}} \ll \Delta t, \Delta l/v$) ⇒ derivative exp^n for q^μ, $\tau^{\mu\nu}$, and ν^μ
[1st order, in Landau frame - fixing field-definition freedom]

$$\mathcal{E} = \epsilon, \quad q^\mu = 0, \quad \mathcal{P} = p - \zeta \nabla \cdot u$$
$$\tau^{\mu\nu} = -\eta \nabla^{\langle\mu} u^{\nu\rangle}, \quad \nu^\mu = -\sigma \Delta^{\mu\nu} \left(E_\nu - T\partial_\nu(\mu/T)\right)$$

Lead to dissipation

- Constrained (conventionally) by requiring local 2nd law [Landau-LIfschitz]

$$TS^\mu = pu^\mu - \mu J^\mu - u_\nu T^{\mu\nu} + \mathcal{O}(\partial)$$

Captures basic features of QFT ➠ should follow directly from an EFT

Thermodynamic response

- To unpack these constraints, it is useful to separate the constitutive tensors into two categories ($J \to J^\mu$ or $T^{\mu\nu}$):

$$J^\mu = J^\mu_{\rm eq} + J^\mu_{\rm hydro}$$

$$\langle JJ\rangle(\omega \to 0, k = 0) \quad \Rightarrow \quad \text{transport coeffs}$$

$$\boxed{\langle JJ\rangle(\omega = 0, k \to 0) \quad \Rightarrow \quad \text{thermo response coeffs}}$$

Many of these coefficients are constrained by matching to *hydrostatic* constitutive relations in global equilibrium

Equilibrium generating functional

- Constraints on thermodynamic response parameters follow directly from the generating function W for euclidean correlators
  - *local* due to screening ➠ derivative expansion

- Introduce a timelike Killing vector β^μ specifying the equilibrium state, and rotate to Euclidean time $\mathbb{R}^3 \times S^1_\beta$

$$\mathcal{L}_\beta g_{\mu\nu} = \mathcal{L}_\beta A_\mu = 0$$

- These conditions impose constraints on the allowed tensors built from T, μ, u^μ, etc

Hydrostatic matching

- Thermodynamic response parameters (e.g. chiral conductivities) are determined by the properties of the equilibrium state [Jensen et al '12, Banerjee et al '12]

$$T^{\mu\nu}_{\text{hydro}}, J^{\mu}_{\text{hydro}} \xleftarrow{\quad ? \quad} W[g_{\mu\nu}, V_\mu]$$

Hydrostatic matching

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$$
\begin{array}{ccc}
T^{\mu\nu}_{\rm hydro}, J^{\mu}_{\rm hydro} & \xleftarrow{\;?\;} & W[g_{\mu\nu}, V_\mu] \\
\Big\downarrow \text{equil state} & & \Big\downarrow W_{\rm eq} \\
T^{\mu\nu}_{\rm eq}, J^{\mu}_{\rm eq} & \longleftrightarrow & \langle T^{\mu\nu}\rangle_{\rm eq} = 2\dfrac{\delta W_{\rm eq}}{\delta g^{\mu\nu}},\quad \langle J^{\mu}\rangle_{\rm eq} = \dfrac{\delta W_{\rm eq}}{\delta V^{\mu}}
\end{array}
$$

Almost all constraints on the constitutive relations can be determined in this way (except 1st order dissipative terms) [Bhattacharyya ’13, Haehl et al ’15]

constraints from matching (in a suitable thermodynamic “frame”)

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Coeffs leading to dissipation are unconstrained by the equilibrium generating functional (the corresponding tensor structures vanish)

➠ can expand about equilibrium using these "dissipative tensors"…

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$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + P_1^{\mu\nu\rho\sigma} \mathcal{L}_\beta g_{\rho\sigma} + \mathcal{O}(\delta^2),$$

[Kovtun & AR, to appear]

$$\mathcal{L}_\beta g_{\rho\sigma} \equiv 2\nabla_{(\rho}\beta_{\sigma)} = \frac{2\dot{T}}{T^2} u_\rho u_\sigma + \frac{2\nabla \cdot u}{3T} \Delta_{\rho\sigma} - \frac{2}{T} q^T_{(\rho} u_{\sigma)} + \frac{2}{T} \sigma_{\rho\sigma},$$

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$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + P_1^{\mu\nu\rho\sigma}\mathcal{L}_\beta g_{\rho\sigma} + P_2^{\mu\nu\rho}\tilde{\mathcal{L}}_\beta A_\rho + \mathcal{O}(\delta^2)$$
$$J^\mu = J^\mu_{\rm eq} + P_3^{\mu\rho\sigma}\mathcal{L}_\beta g_{\rho\sigma} + P_4^{\mu\rho}\tilde{\mathcal{L}}_\beta A_\rho + \mathcal{O}(\delta^2),$$

[Kovtun & AR, to appear]

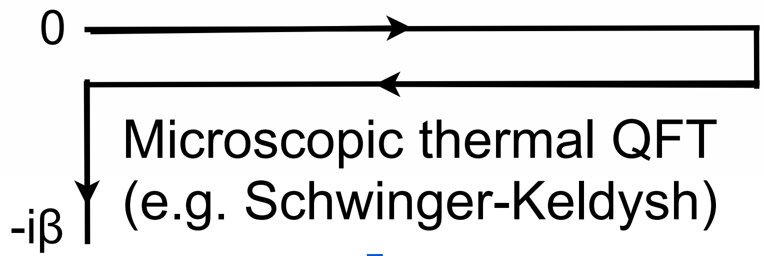
$$\mathcal{L}_\beta g_{\rho\sigma} \equiv 2\nabla_{(\rho}\beta_{\sigma)} = \frac{2\dot{T}}{T^2}u_\rho u_\sigma + \frac{2\nabla\cdot u}{3T}\Delta_{\rho\sigma} - \frac{2}{T}q^T_{(\rho}u_{\sigma)} + \frac{2}{T}\sigma_{\rho\sigma}$$

$$\tilde{\mathcal{L}}_\beta A_\rho \equiv \mathcal{L}_\beta A_\rho + \partial_\rho\Lambda_\beta = u_\rho\left(\frac{\dot{\mu}}{T}\right) - \frac{1}{T}q^J_\rho$$

where

$$q^\rho_T = \Delta^{\rho\sigma}\left(\dot{u}_\sigma + \partial_\sigma \ln T\right)$$
$$q^J_\rho = \Delta_{\rho\sigma}\left(E^\sigma - T\partial^\sigma\left(\frac{\mu}{T}\right)\right)$$

Hydrodynamic EFT



Microscopic thermal QFT
(e.g. Schwinger-Keldysh)

$$\phi_r = \frac{1}{2}(\phi_1 + \phi_2)$$

$$\phi_a = \phi_1 - \phi_2$$

local thermal equilibrium

$$\frac{\partial}{T} \ll 1$$

➠ What EFT incorporates both *dissipation* and *fluctuations?*

[e.g. Adler & Wainwright '70]

Impact seen through e.g. long-time tails

$$\langle v(t)v(0)\rangle \sim \frac{1}{t^{d/2}}$$

Hydrodynamic EFT

Classical hydrodynamics

Thermal fluctuations

Linear Response - Eg. diffusion

$$G^{\text{ret}}_{nn} = -i\langle n_r n_a\rangle = \frac{D\chi\vec{k}^2}{i\omega - D\vec{k}^2},\quad G^{\text{adv}}_{nn} = -i\langle n_a n_r\rangle = \frac{-D\chi\vec{k}^2}{i\omega + D\vec{k}^2},\quad G^{\text{sym}}_{nn} = 2\langle n_r n_r\rangle = \frac{4TD\chi\vec{k}^2}{\omega^2 + (D\vec{k}^2)^2}$$

from hydro (diffusion eqn)

factor of T from FDT

What is the (nonlocal) effective action that leads to all these correlators in an *equilibrium* state?

➠ “bottom up” approach - add auxiliary fields (φ_a), background sources ($g_{\mu\nu}$, A_μ) etc [Harder, Kovtun & AR ’15]

$$Z[A_r, A_a] = \int [D\phi_r][D\phi_a] e^{iS[\phi_r,\phi_a,A_r,A_r]}$$

$$\xi^a_\mu = A^a_\mu + \partial_\mu \phi^a$$

hydro current

$$S = \int d^4x \left[\xi^a_\mu (J^\mu_{\text{cl}}[\phi_r, A_r] + iT\sigma\Delta^{\mu\nu}\xi^a_\nu)\right]$$

➠ leads to all $\langle J_\mu J_\nu\rangle^{R,A,S}$

Linear response - Eg. neutral fluid

Similarly, one can construct an effective action to compute the E-M tensor correlators in linearized hydrodynamics $\langle T_{\mu\nu} T_{\rho\sigma} \rangle^{R,A,S}$

[Harder, Kovtun & AR '15]

$$Z[g_r, g_a] = \int [D\phi^r_\mu][D\phi^a_\mu] e^{iS[\phi^r, \phi^a, g_r, g_a]}$$

$$\xi^a_{\mu\nu} = g^a_{\mu\nu} - 2\nabla_{(\mu}\phi^a_{\nu)}$$

hydro E-M tensor

$$S = \int d^4x \sqrt{-g_r} \left[\xi^a_{\mu\nu} \left(T^{\mu\nu}_{\rm cl}[\phi_r, g_r] + iTG^{\mu\nu\rho\sigma}\xi^a_{\rho\sigma}\right)\right]$$

FDT relates the tensor structure and coefficient to the dissipative terms in $T_{\mu\nu}$

What symmetry structure is behind this effective theory?

Symmetry proposal (microscopic SK formalism)

$$Z[j_1,j_2] = \int d\tilde{q}_1 d\tilde{q}_2 dq_f \langle \tilde{q}_1|\rho|\tilde{q}_2\rangle \int_{\tilde{q}_1}^{q_f} Dq_1 \int_{\tilde{q}_2}^{q_f} Dq_2 e^{iS[q_1,j_1]-iS[q_2,j_2]}$$

$$S[q_1,j_1] - S[q_2,j_2] = \int q_a \mathrm{EOM}(q_r, j_r) + \mathcal{O}(j_a, q_a^2)$$

Structure observed in "bottom up" effective action!
$\mathrm{EOM} \to \nabla_\mu T^{\mu\nu} = 0$

- To manifest symmetries in the SK formalism, need (1,2) fields
- BUT - hydrodynamics involves only the r-fields
  - propose a nonlinear realization in the a-sector

$$G_1 \times G_2 \to G_r$$

[Harder, Kovtun & AR '15]

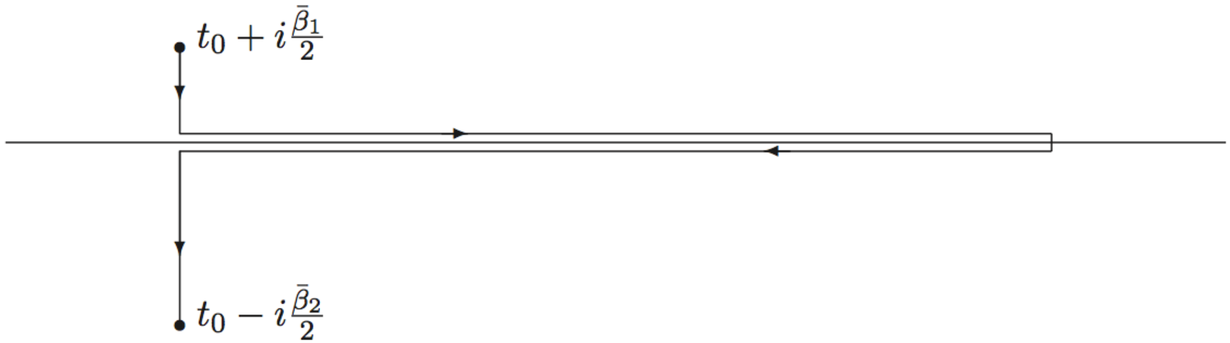
  - G_r manifest in hydrodynamics, while the auxiliary fields required to localize the action do indeed couple as Goldstone bosons…

linear G_a invariants
$$\begin{cases} \xi^a_{\mu\nu} = g^a_{\mu\nu} - 2\nabla_{(\mu}\phi^a_{\nu)} \\ \xi^a_\mu = A^a_\mu + \partial_\mu \phi^a \end{cases}$$

dynamical fields

(Local) KMS condition

[Kovtun & AR, to appear]



$$\beta^\mu = \frac{1}{2}(\beta_1^\mu + \beta_2^\mu)$$

$$\varphi_a^\mu = \beta_1^\mu - \beta_2^\mu$$

$$W[g^1_{\mu\nu}(x), g^2_{\alpha\beta}(x)] = W\left[g^1_{\mu\nu}\left(-t + \frac{i\beta_1}{2}, -\vec{x}\right), g^2_{\alpha\beta}\left(-t - \frac{i\beta_2}{2}, -\vec{x}\right)\right]$$

KMS constraint on full generating functional (implying thermal equilibrium state)

[see also Crossley et al '15]

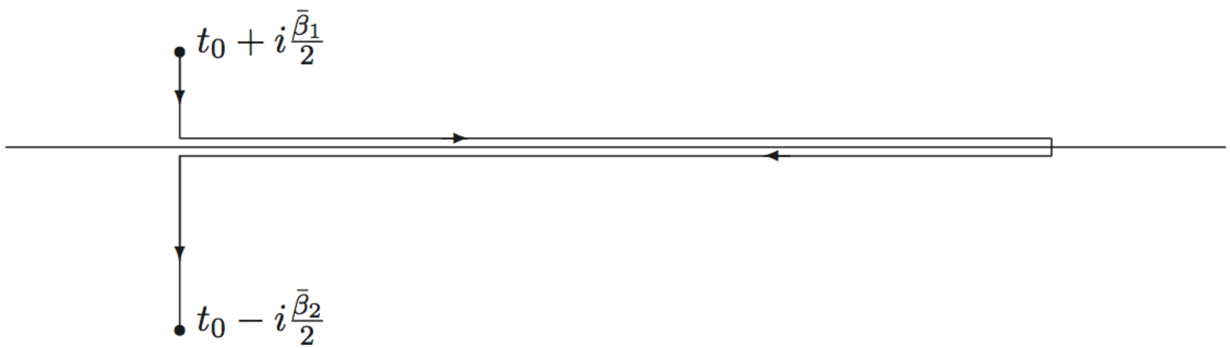
Consider infinitesimal form...

$$g^a_{\mu\nu}(-x) + i\mathcal{L}_\beta g^r_{\mu\nu}(-x) + i\mathcal{L}^a_{\varphi^a} g^a_{\mu\nu}(-x) + \cdots = 0$$

Reduces to existence of Killing vector if a-sector fields vanish, but in general links *dissipative* terms to *fluctuations*

(Local) KMS condition

[Kovtun & AR, to appear]



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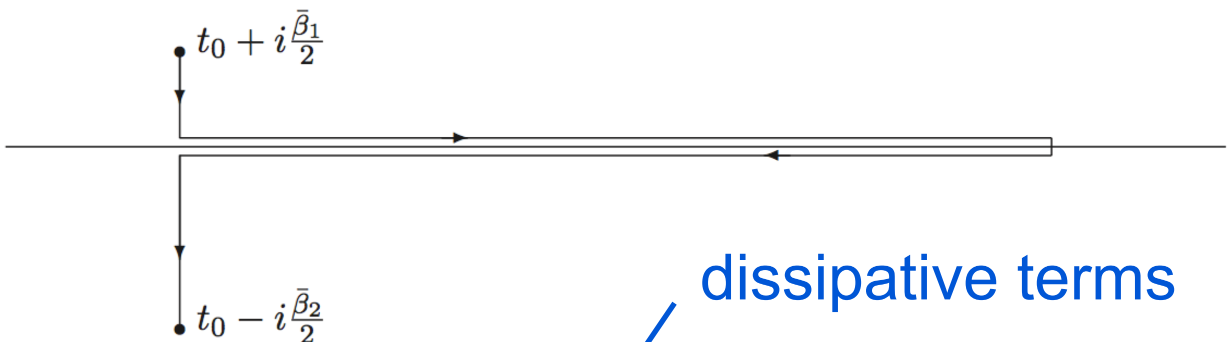
Natural expansion parameter about equilibrium...?

Effective action - e.g. neutral fluid

[Kovtun & AR, to appear]



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dissipative terms

$$S_{\text{eff}} = \int \tfrac{1}{2}\sqrt{-g^r}\left[\mathcal{L}_{\text{eq}} + \mathcal{L}^{(1)} + \cdots\right],$$

$$\mathcal{L}^{(1)} = \frac{i}{2}P_1^{\mu\nu\alpha\beta}\eta^a_{\mu\nu}\eta^a_{\alpha\beta} + \mathcal{O}(\partial^2, a^3) \qquad \eta^a_{\mu\nu} = \xi^a_{\mu\nu} - i\mathcal{L}_\beta g_{\mu\nu}$$

$$= P_1^{\mu\nu\alpha\beta}\xi^a_{\mu\nu}\left(\mathcal{L}_\beta g_{\alpha\beta} + \frac{i}{2}\xi^a_{\alpha\beta}\right) + \cdots$$

Structure required to reproduce all 2-pt functions

Coefficients linked by KMS condition, i.e. FDT

Eqn of motion for φ^a

$$\nabla_\mu(T^{\mu\nu}_{\text{eq}} + T^{\mu\nu}_{(1)}) = 0 \qquad T^{(1)}_{\mu\nu} = P_1^{\mu\nu\alpha\beta}\mathcal{L}_\beta g_{\alpha\beta} + \cdots$$

Concluding Remarks

Towards a universal hydrodynamic EFT

- dissipative terms in 1st order hydro are efficiently captured by expanding around equilibrium with L_β(source)=0

- This structure extends to form a low energy effective action incorporating fluctuations, that (at the linear level) appears consistent with the SK formalism for thermal QFT

  - symmetries for the doubled degrees of freedom are realized in a mixed phase, with a-sector nonlinearly realized
  - the KMS condition for thermal equilibrium is naturally enhanced to a local translational symmetry

  - NB: Related work, at nonlinear order [Haehl et al ’15, Crossley et al ’15], suggests TQFT structure in the a-sector, important in the hydro limit. For earlier work see [Endlich et al ’12], ...

- Further work:

  - explicit realization of the symmetry pattern
  - applications - fluctuation-induced shift to transport coefficients
  - extension to 2+1D with non-dissipative transport (Hall viscosity)